TM-70-1022-15

TECHNICAL MEMORANDUM

UNCONTROLLED MOTION OF A SPINNING SPACECRAFT

Bellcomm

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W., WASHINGTON, D.C. 20024

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Uncontrolled Motion of a Spinning Spacecraft

TM- 70-1022-15

DATE- September 3, 1970

FILING CASE NO(S)- 620

AUTHOR(S)- R. J. Ravera

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ABSTRACT

At the beginning of a proposed artificial gravity experiment, the spin vector of Skylab B would be aligned with the sun vector so that the vehicle's solar arrays produce maximum power. If not controlled, gravity gradient torque and the apparent solar motion due to the earth's yearly rotation about the sun would cause the spin vector to move away from the sun. This motion is studied on time bases equivalent to one orbit and to the whole 30-day duration of the proposed experiment. It is shown that if the experiment is conducted during a judiciously chosen time and without any active control, the motion causes a maximum power loss of 2.2%. It would require approximately 450 lbs of RCS fuel and daily crew involvement to perform control maneuvers to keep the spin vector pointed exactly at the sun and eliminate this small power loss.

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ABSTRACT ONLY

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SUBJECT: Uncontrolled Motion of a Spinning Spacecraft - Case 620

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FROM: R. J. Ravera

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TECHNICAL MEMORANDUM

1.0 INTRODUCTION

An artificial gravity experiment is being considered for the proposed second Skylab. The artificial g field is to be obtained by spinning the vehicle about its mass center. As the Skylab B configurations under study include planar solar arrays perpendicular to the vehicle Z-axis, it is desirable that the vehicle Z-axis be the spin axis and that it be aligned parallel to the solar vector (see Figure 1). However, the vehicle Z-axis is not necessarily the axis of maximum moment of inertia (X_3-axis), and rotational stability requires that the spin axis be the X_3 axis. With the addition of ballasting beams to the Skylab A configuration, the X_3-axis can be brought to within about 5° of the vehicle Z-axis.[1] Other Skylab B configurations under study (without the ATM) can, with careful attention to mass distribution, achieve similar alignment.

The spin angular momentum vector can be initially aligned with the X_3-axis and with the solar vector, and the power penalty is almost negligible. But there are several effects which tend to move the X_3-axis off the sun line. These are gravity gradient torques, aerodynamic torques, and the motion of the earth about the sun. An estimate of CSM RCS fuel required to counter the effects of the bias gravity gradient torque and apparent sun motion has been given in Reference 2. Such fuel estimates are important if it becomes necessary to control the motion on a daily basis, regardless of the magnitude of the X_3-axis excursion from the sun line. The purposes of this study were to obtain a precise description of uncontrolled vehicle motion over both short and long terms and to assess the feasibility of using passive precession control by conducting the artificial g experiment during a pre-selected, optimum time.

2.0 COORDINATE SYSTEMS

Because the vehicle is spinning, it is desirable to carry out the analysis in a body fixed, principal axis frame (X_1, X_2, X_3), called spacecraft (sc) coordinates. To study the

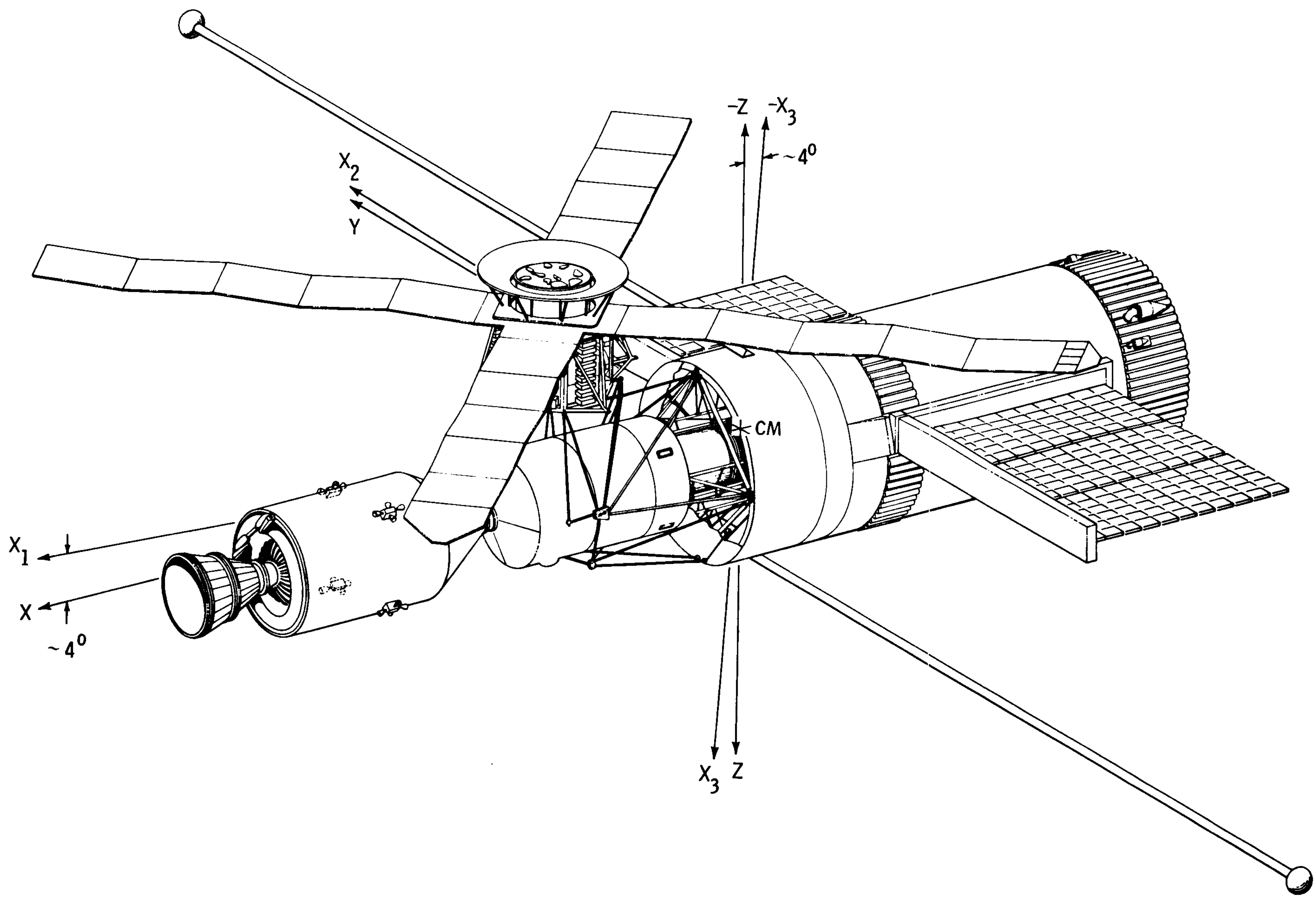

FIGURE 1 - SKYLAB B WITH BALLAST FOR ARTIFICIAL GRAVITY EXPERIMENT

motion of the vehicle relative to the sun, we introduce the solar inertial coordinate system (X_{si}, Y_{si}, Z_{si}), which is shown in Figure 2(a). Also shown in Figure 2(a) is the nodal coordinate system (X_n, Y_n, Z_n), related to the solar inertial frame by solar pointing angles β and ψ and to the local vertical system ($X_{\ell v}$, $Y_{\ell v}$, $Z_{\ell v}$) through the angle η. The relationships of β and ψ to launch date and time, orbital parameters and time after launch are well established.[3] The angle η, measured from the ascending node, is found from

$$\eta = \omega_0 t \tag{1}$$

where ω_0 is the orbital rate and t is the time after a nodal pass. To go from the solar inertial coordinate system to sc coordinates an Euler angle transformation (Y X Z-sequence) is employed. The Euler angles θ_1, θ_2, and θ_3 are pictured in Figure 3. A vector in local vertical coordinates, represented by a 3 × 1 matrix $u^{\ell v}$, is expressible in sc coordinates by

$$u^{sc} = T^Z_{\theta_3} \; T^X_{\theta_1} \; T^Y_{\theta_2} \; T^X_{\beta} \; T^Y_{(\psi-\eta)} u^{\ell v} \tag{2}$$

where

$$T^X_\zeta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\zeta & s\zeta \\ 0 & -s\zeta & c\zeta \end{bmatrix}$$

$$T^Y_\zeta = \begin{bmatrix} c\zeta & 0 & -s\zeta \\ 0 & 1 & 0 \\ s\zeta & 0 & c\zeta \end{bmatrix}$$

and

$$T^Z_\zeta = \begin{bmatrix} c\zeta & s\zeta & 0 \\ -s\zeta & c\zeta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

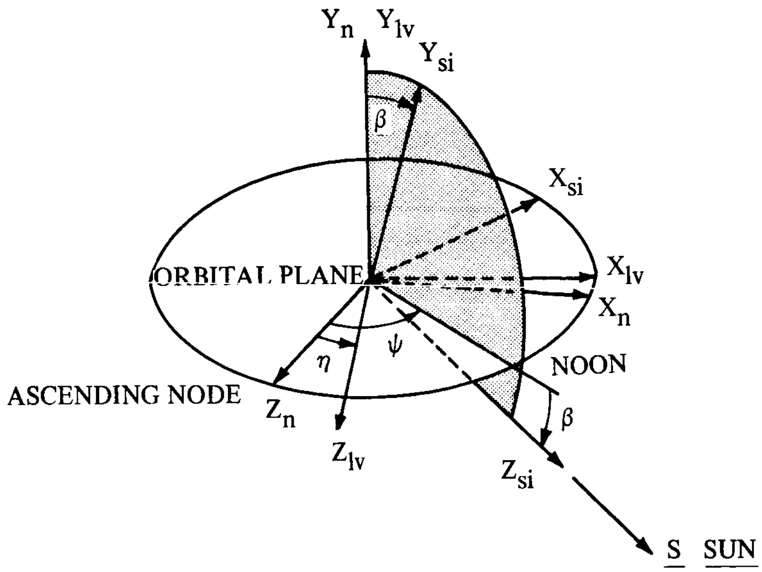

FIGURE 2 (a) - COORDINATE SYSTEMS

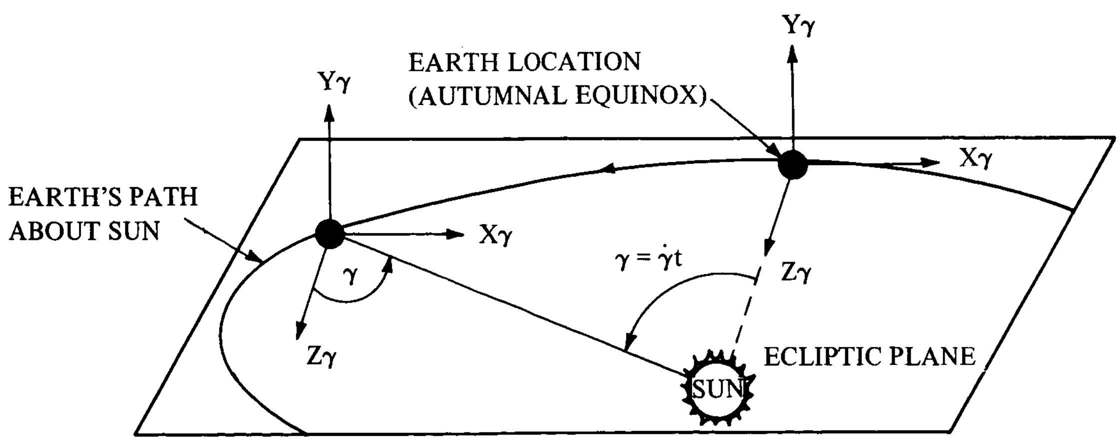

FIGURE 2 (b) - GEOCENTRIC INERTIAL COORDINATE SYSTEM Xγ Yγ Zγ

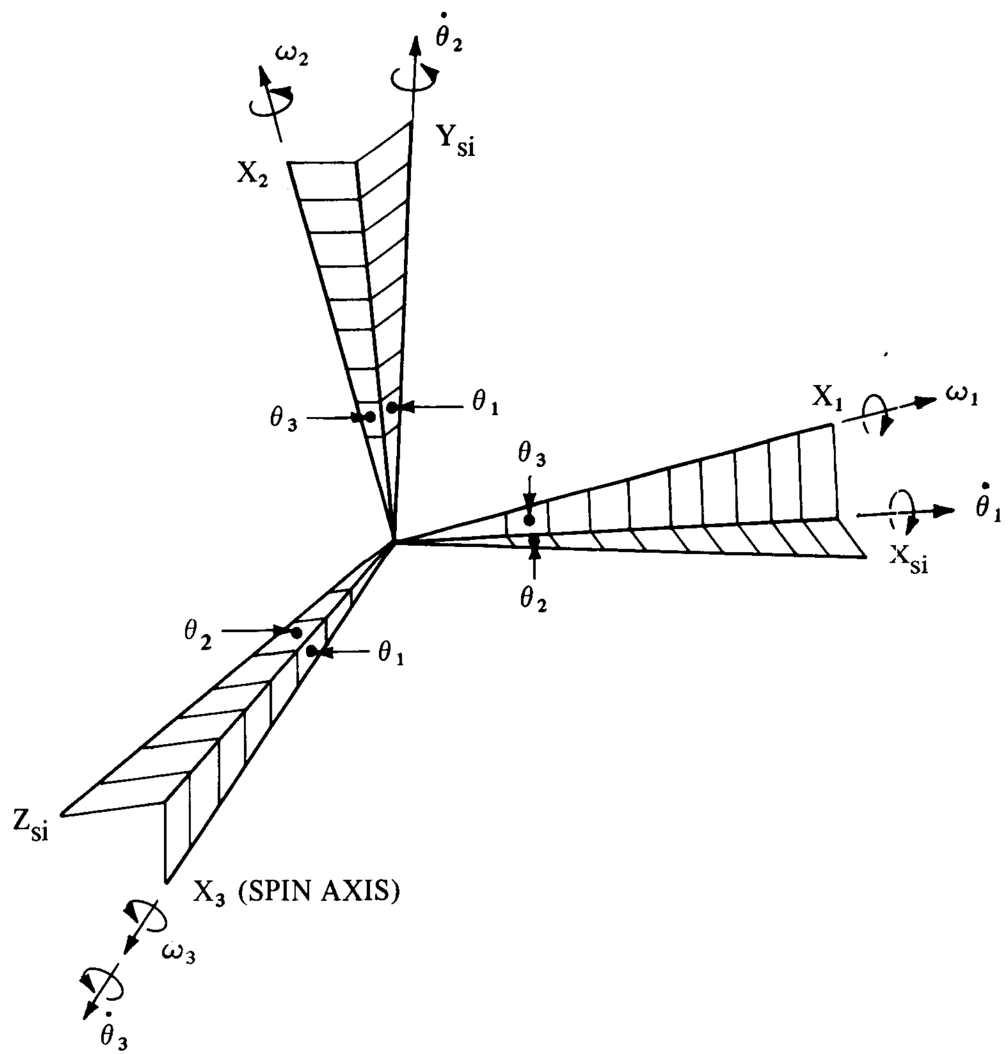

FIGURE 3 - RELATIONSHIP OF SPACECRAFT COORDINATES TO SOLAR INERTIAL COORDINATES AND SPACECRAFT BODY RATES TO EULER ANGLE RATES

Similarly, a vector in the nodal system is expressible in sc coordinates by

$$u^{sc} = T^Z_{\theta_3} \; T^X_{\theta_1} \; T^Y_{\theta_2} \; T^X_{\beta} \; T^Y_{\psi} \; u^n \qquad (3)$$

We define a geocentric inertial frame (X_γ, Y_γ, Z_γ) whose Z-axis is directed toward the sun at autumnal equinox (see Figure 2(b)). A vector in the nodal frame is related to a vector in this inertial frame by

$$u^n = T^Z_i \; T^Y_{\Omega_r} \; T^Z_e \; u^\gamma \qquad (4)$$

where i is the orbit inclination, Ω_r is the angle between ascending node and Z_γ, and e is the angle between the ecliptic and equatorial planes.

The Euler angle rates, $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$ are related to the spacecraft body rates relative to the solar inertial frame, $(\omega^{sc}_{sc/si})_1$, $(\omega^{sc}_{sc/si})_2$, and $(\omega^{sc}_{sc/si})_3$ by the matrix representation

$$\dot{\theta} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ \dfrac{s\theta_3}{c\theta_1} & \dfrac{c\theta_3}{c\theta_1} & 0 \\ s\theta_3 t\theta_1 & c\theta_3 t\theta_1 & 1 \end{bmatrix} \omega^{sc}_{sc/si} \qquad (5)$$

Angles θ_1 and θ_2 are defined, respectively, as the nutation and precession angles of the spacecraft with respect to the solar inertial system.

3.0 DISTURBANCE TORQUES

3.1 Aerodynamic Torque

Over an orbit, the orientation of the vehicle spin vector will not change significantly; specifically, results show that the maximum precession per orbit is on the order of 0.2 degrees and nutation half-an-order-of-magnitude less. Because of this almost fixed orientation, the per-orbit average of aerodynamic torque tends to zero. Consequently, aerodynamic torques are not a significant factor in the precession and nutation of the vehicle. As a matter of interest, a calculation has shown that over a thirty day period, aerodynamic drag forces which constantly oppose vehicle spin have no significant effect on the spin rate.

3.2 Gravity Gradient Torque

Gravity gradient torque is the major disturbing torque on the vehicle. The expression for gravity gradient torque in local vertical coordinates is

$$T_{gg}^{\ell v} = 3\omega_0^2 \, \tilde{\rho}^{\ell v} \, I^{\ell v} \, \rho^{\ell v}$$

where I is the inertia matrix of the vehicle and $\rho^{\ell v}$ is the unit vector along the local vertical; that is,

$$\rho^{\ell v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The tilde operator ($\sim$) over the matrix representation of a vector is isomorphic to the vector cross product. For example, consider the vectors

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

and

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

The tilde operator on u designates

$$\tilde{u} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

and

$$\tilde{u}v = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{bmatrix}$$

It is clear that the terms in the matrix resulting from $\tilde{u}v$ are the components of the vector cross product $\underline{u} \times \underline{v}$.

In sc coordinates, the gravity gradient torque is

$$T_{gg}{}^{sc} = 3\omega_0^2 \, \tilde{\rho}^{sc} \, I^{sc} \, \rho^{sc} \tag{6}$$

where, from (2),

$$\rho^{sc} = T^Z_{\theta_3} \, T^X_{\theta_1} \, T^Y_{\theta_2} \, T^X_\beta \, T^Y_{(\psi-n)} \, \rho^{\ell v}$$

4.0 APPARENT SOLAR MOTION

The annual rotation of the earth about the sun causes an approximate 1 degree/day rotation of the sun in the geocentric inertial frame (γ). Since the equations of motion will be based on measurements made with respect to the geocentric inertial frame, the apparent solar motion will be naturally included.

5.0 EQUATIONS OF MOTION

The equations of motion of the vehicle in spacecraft coordinates are, in matrix form,

$$\overset{\circ}{H}{}^{sc} + \tilde{\omega}_T^{sc} H^{sc} = T_{gg}{}^{sc} \tag{7}$$

where H^{sc} is the total angular momentum of the vehicle expressed in sc coordinates, $\overset{\circ}{H}{}^{sc}$ is the time rate of change of H^{sc} computed with respect to sc coordinates, and $\omega_T{}^{sc}$ is the total angular velocity of the vehicle with respect to the geocentric inertial frame, written in sc coordinates.

Thus,

$$H^{sc} = I\omega_T^{sc} \tag{8}$$

and

$$\overset{\circ}{H}{}^{sc} = I\dot{\omega}_T^{sc} \tag{9}$$

The total angular velocity ω_T^{sc} can be written

$$\omega_T^{sc} = \omega_{sc/si}^{sc} + \omega_{si/\gamma}^{sc} \tag{10}$$

where $\omega_{sc/si}^{sc}$ is the angular velocity of the spacecraft with respect to the solar inertial frame and $\omega_{si/\gamma}^{sc}$ is the angular velocity of the solar inertial frame with respect to the geocentric inertial frame. From (3) and (4), noting that i and e are fixed,

$$\omega_{si/\gamma}^{sc} = \dot{\Omega}_r^{sc} + \dot{\beta}^{sc} + \dot{\psi}^{sc} \tag{11}$$

Expressions for $\dot{\Omega}_r$, $\dot{\beta}$, and $\dot{\psi}$ may be found in Reference 3. They are transformed into sc coordinates by using the appropriate transform matrices defined in (2), (3) and (4).

Since it is the motion of the vehicle with respect to the solar inertial frame that is of major interest, we seek to solve for $\omega^{sc}_{sc/si}$. Substituting (8) and (9) into (7) gives

$$I\dot{\omega}^{sc}_{T} + \tilde{\omega}^{sc}_{T} I\omega^{sc}_{T} = T_{gg}^{sc} \tag{12}$$

The total vehicle body rates are found by solving (12) for ω^{sc}_{T}; formally we can write

$$\omega^{sc}_{T} = \int_0^t \dot{\omega}^{sc}_{T} dt = I^{-1}\int_0^t \left[T_{gg}^{sc} - \tilde{\omega}^{sc}_{T} I\omega^{sc}_{T}\right] dt \tag{13}$$

Equation (10) gives

$$\omega^{sc}_{sc/si} = \omega^{sc}_{T} - \omega^{sc}_{si/\gamma} \tag{14}$$

Substituting the values of ω^{sc}_{T} and $\omega^{sc}_{si/\gamma}$, determined from (13) and (11) respectively, into (14) gives the desired quantity, $\omega^{sc}_{sc/si}$. The final step is to substitute (14) into (5) so that the Euler angle rates $\dot{\theta}_1$ (nutation) and $\dot{\theta}_2$ (precession) may be solved for and integrated to give the angular displacement of the vehicle from the solar inertial frame.

6.0 RESULTS

The numerical results are based on a vehicle whose mass properties were derived from Skylab A by augmenting it for a one year mission and including ballast for mass properties control.[2] The spin rate is 6 RPM.

The first set of results describe the precise vehicle motion over the short term. Using β angle values of +57.0° and -54.0°, indicated by points 1 and 2 on the β history in Figure 4,

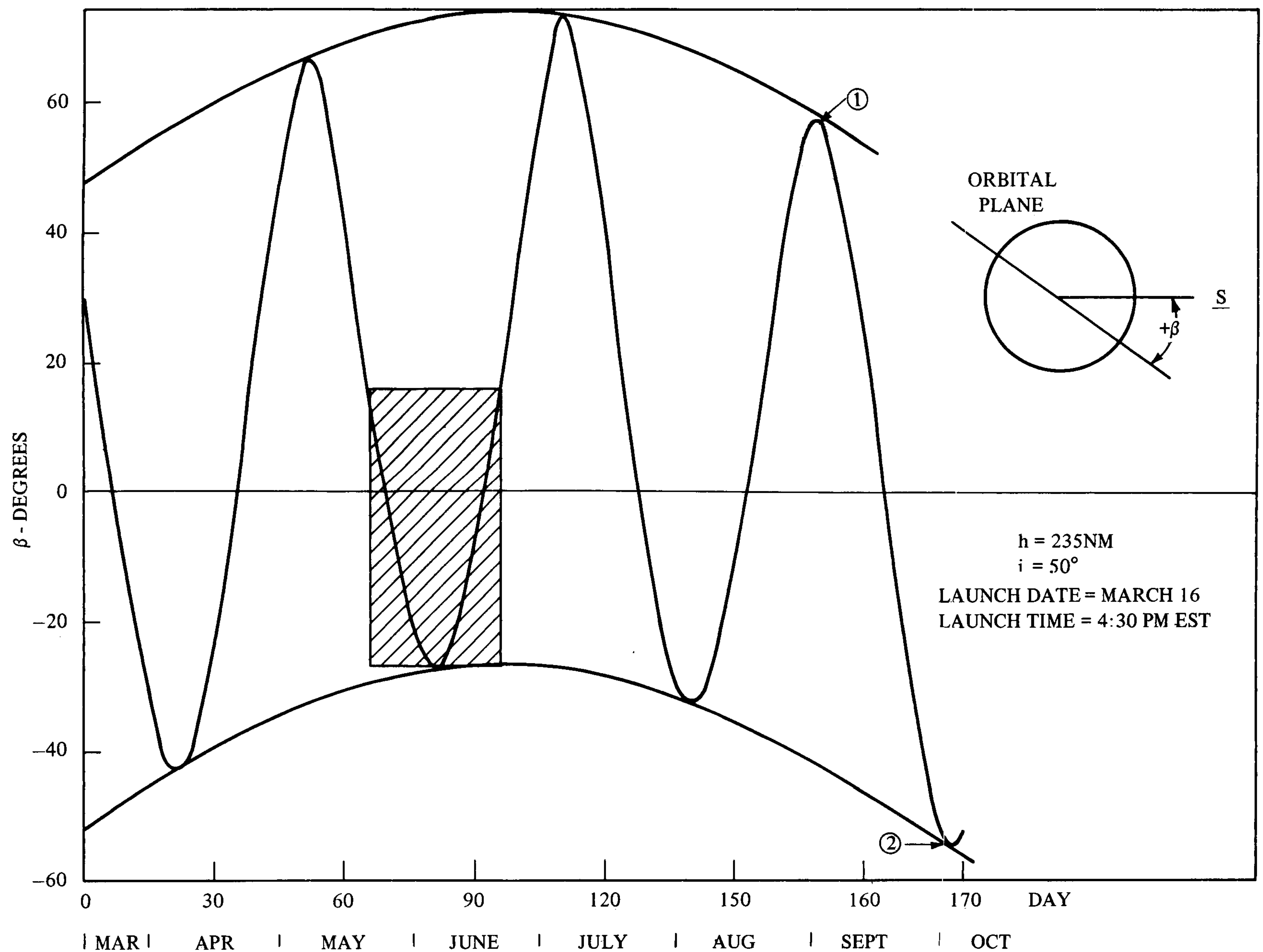

FIGURE 4 - β-ANGLE HISTORY

complete θ_1 vs θ_2 histories for a single orbit were generated. Figures 5 and 6 were derived by neglecting the effect of the apparent solar motion; it is clear that gravity gradient torque produces, over an orbit, a completely precessional motion with nutational excursions returning to zero at the conclusion of the orbit. Figures 7 and 8 include the effect of solar motion. The motion of the spin vector with respect to the sun is still primarily precessional, with only a small amount of residual nutation angle remaining at the end of the orbit. Furthermore, for the direction of the spin chosen (spin angular momentum vector pointing at the sun) it is clear by comparing Figures 5 and 7 and 6 and 8 that solar motion causes greater precessional excursions for positive β and less for negative β, indicating that the possibility exists for optimizing CSM RCS fuel required to control precession. This was, in fact, one of the primary conclusions of Reference 2.

The fuel optimization strategy just mentioned would be considered only if it is necessary and/or desirable to control the spin-vector motion on a daily basis. Suppose the experiment is conducted during a time when there is an optimum β angle trace and the vehicle is left uncontrolled; is the resulting long term motion tolerable? Based on Reference 2, an "optimum" 30-day β history was approximated and is shown in Fig. 4 as the cross hatched area. Beginning about 64 days after launch, the vehicle, spinning at 6 RPM and pointing directly at the sun ($\theta_1 = \theta_2 = 0$), is allowed to move in a completely uncontrolled fashion for 30 days. The results, on a daily basis, are shown in Figure 9. Nutation excursions remain within $-1.41° \leq \theta_1 \leq 2.15°$ while precession excursions remain within $-11.65° \leq \theta_2 \leq +11.12°$. This motion introduces a maximum electrical power penalty of only 2.2%. It therefore seems that passive precession control of the spinning Skylab is possible.

7.0 CONCLUSIONS

This memorandum has demonstrated that the motion of a Skylab vehicle under the action of major external influences undergoes, primarily, a precessional motion with respect to a solar inertial reference.

The results for short term motion support the conclusions of Reference 2 that CSM RCS fuel consumption can be optimized for a daily thruster firing strategy of precession control. More importantly, the results for the long term indicate that if the artificial gravity experiment is conducted during a judiciously chosen time, it appears possible to control precession passively.

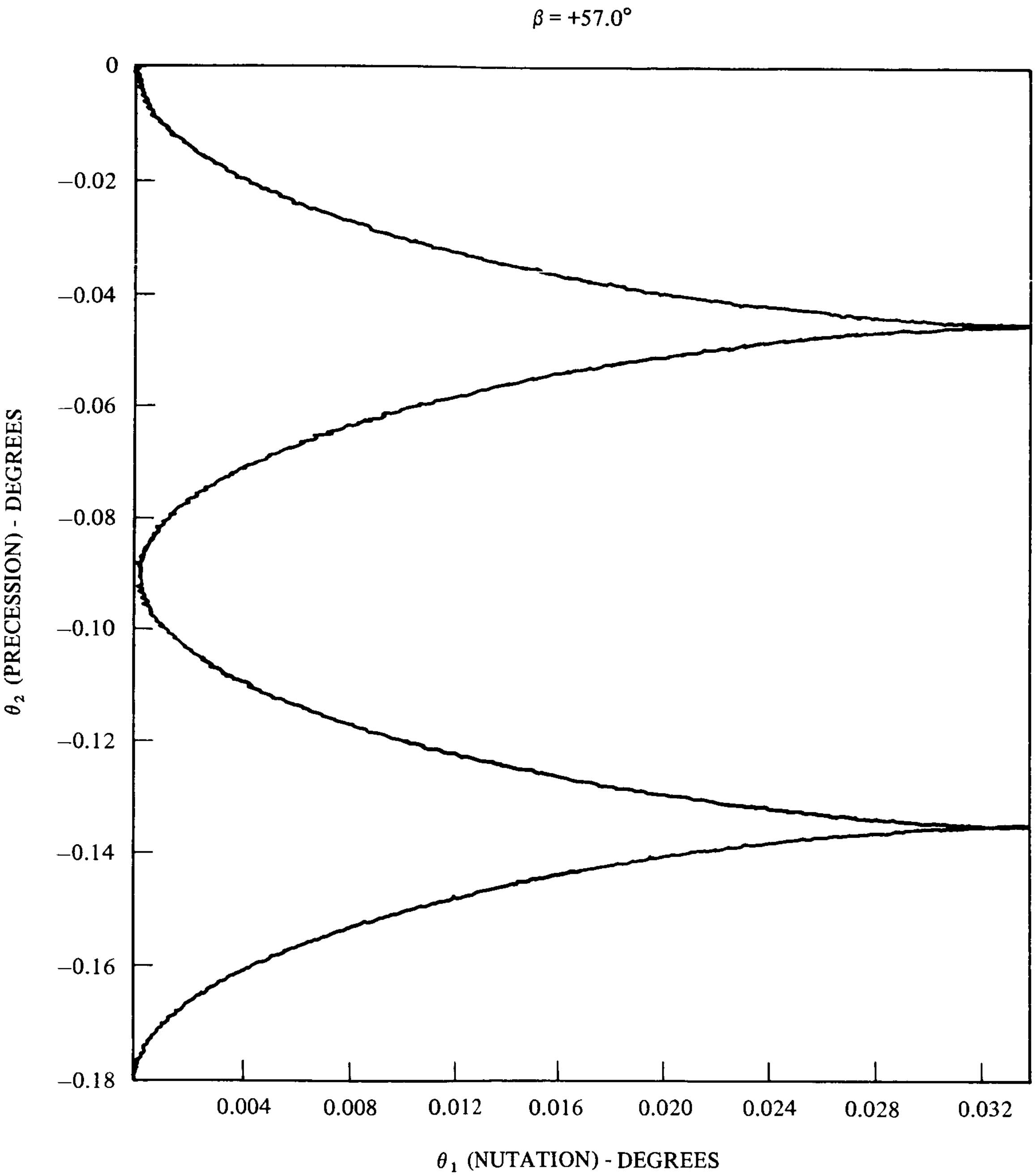

FIGURE 5 - PRECESSION VS NUTATION FOR ONE ORBIT - GRAVITY GRADIENT ONLY

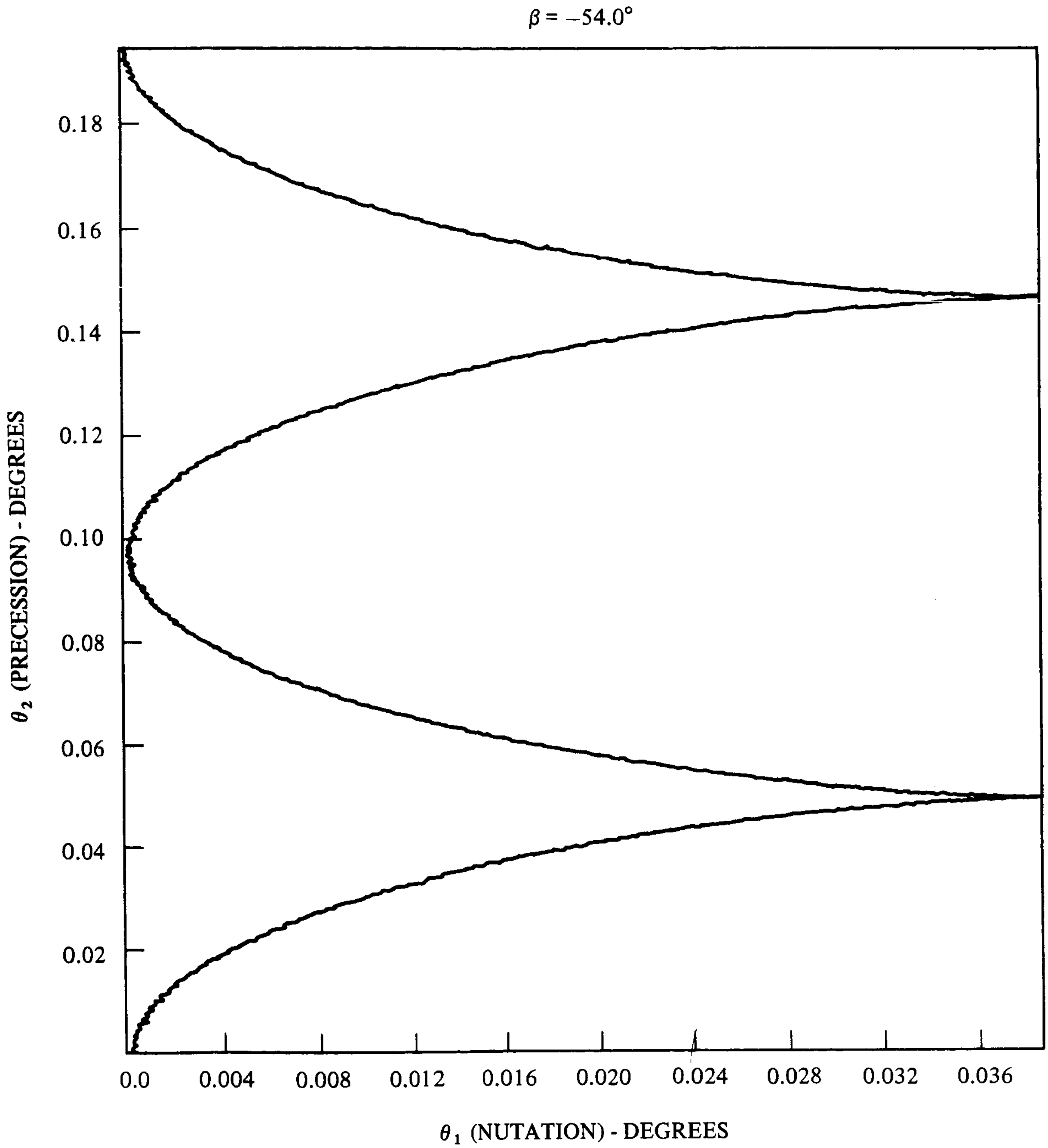

FIGURE 6 - PRECESSION VS NUTATION FOR ONE ORBIT - GRAVITY GRADIENT ONLY

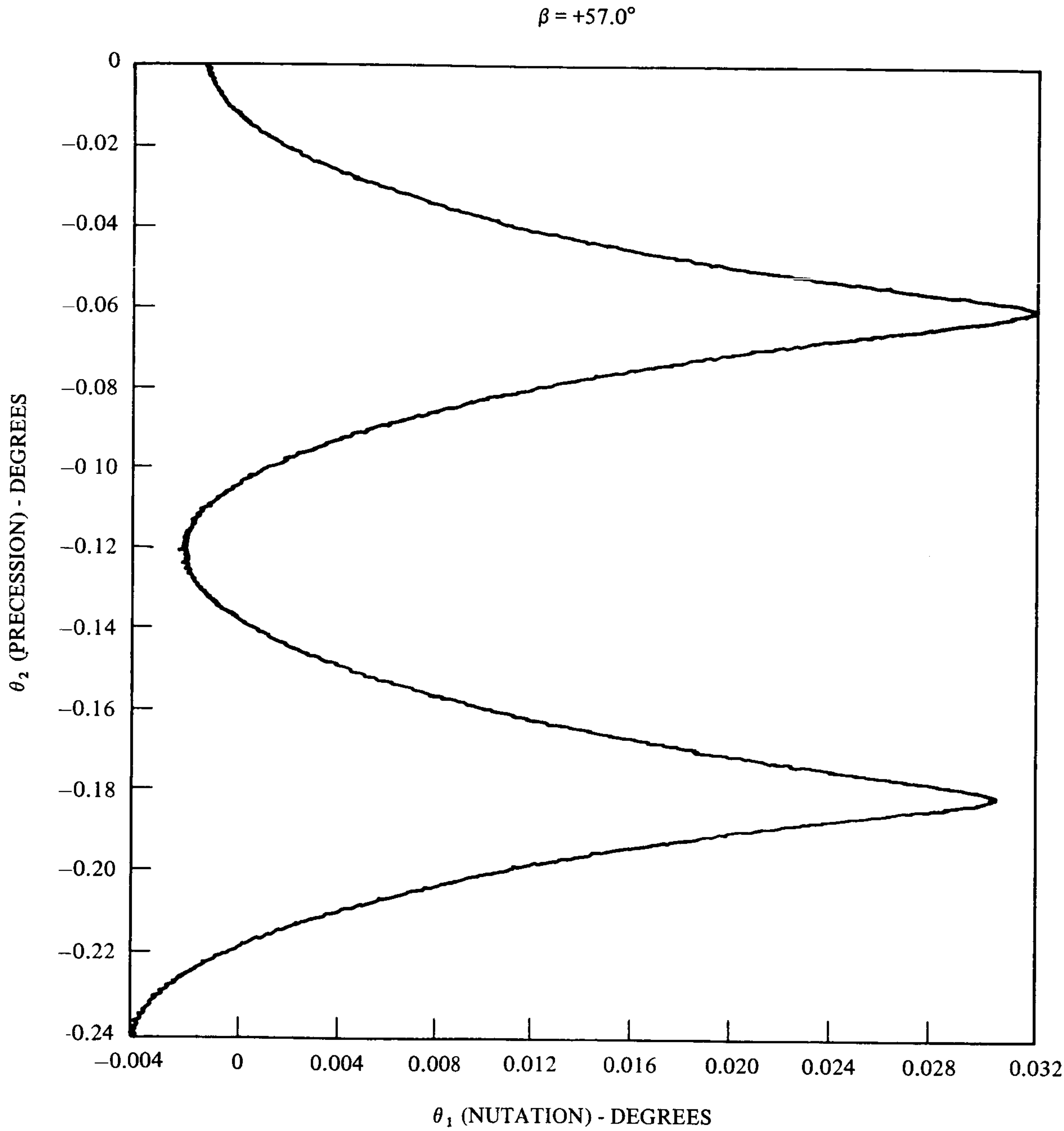

FIGURE 7 - PRECESSION VS NUTATION FOR ONE ORBIT - GRAVITY GRADIENT AND SOLAR PRECESSION

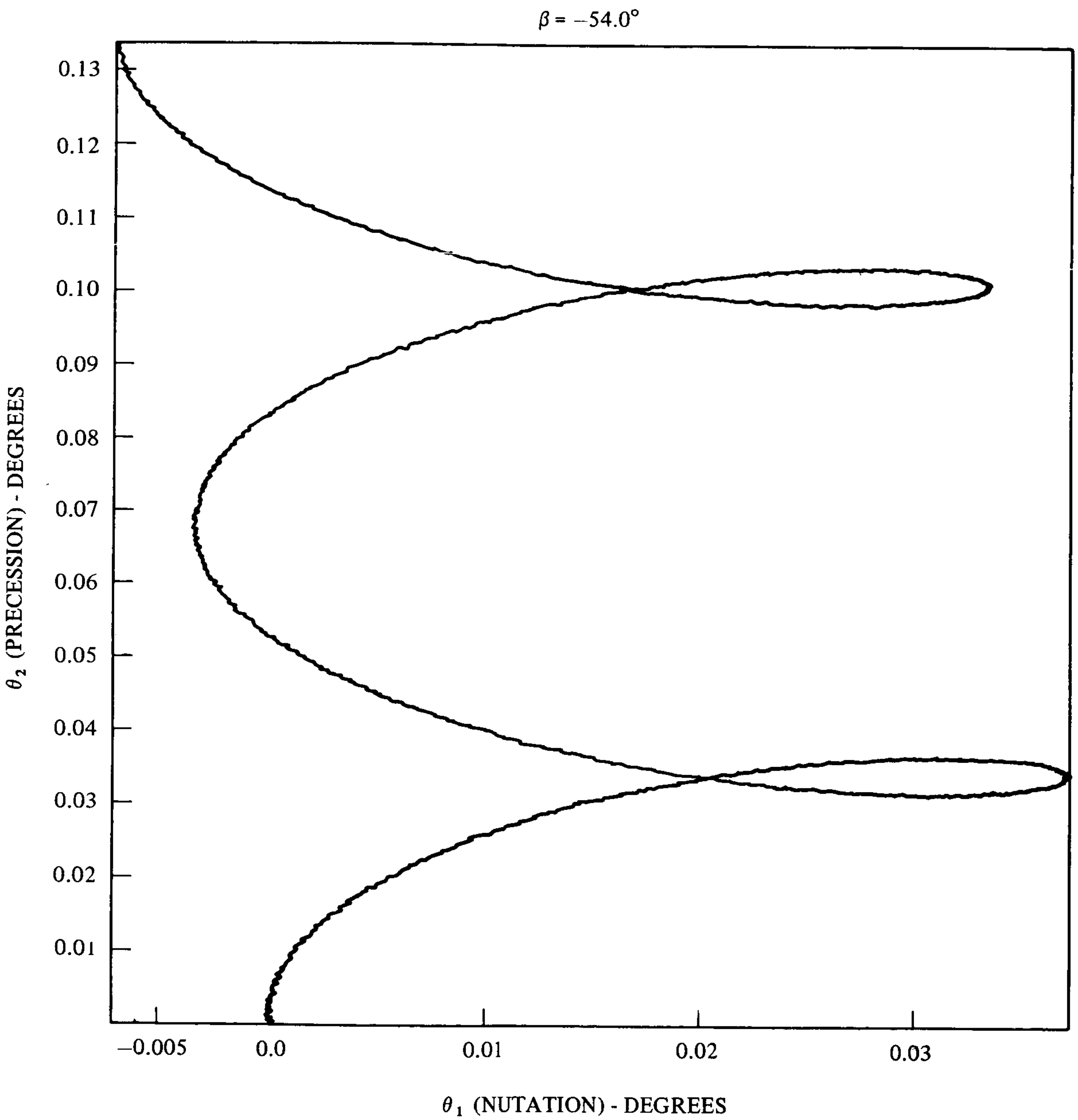

FIGURE 8 - PRECESSION VS NUTATION FOR ONE ORBIT - GRAVITY GRADIENT AND SOLAR PRECESSION

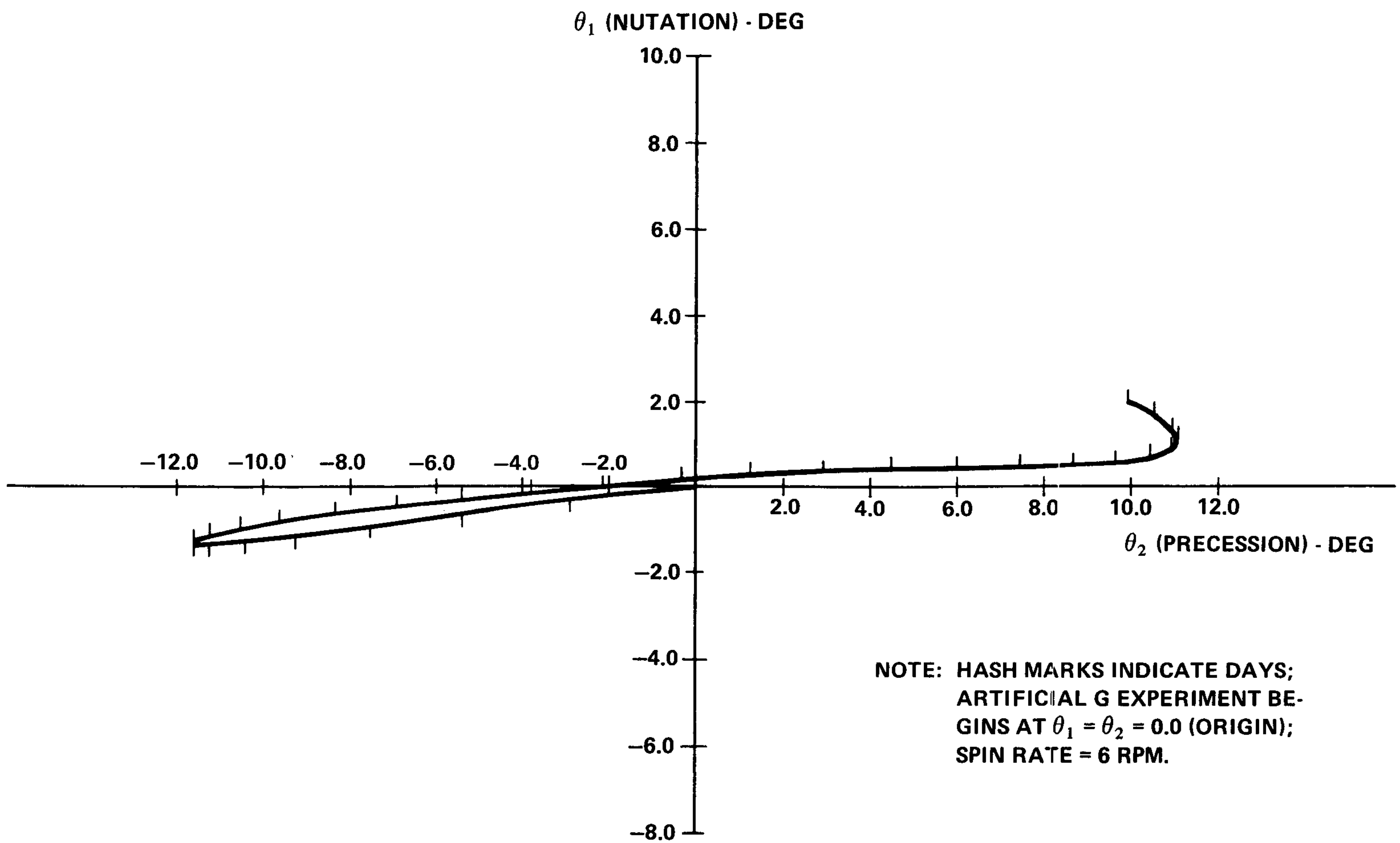

FIGURE 9 - DAILY VALUES OF θ_1 AND θ_2 FOR A 30-DAY ARTIFICIAL GRAVITY EXPERIMENT, CONDUCTED DURING A JUDICIOUSLY CHOSEN TIME.

Even if it is desired to trim the attitude of the vehicle at extreme points of the excursions, such occasions occur only twice during the experiment and precession control is still nearly passive. The CSM RCS fuel weight savings of about 450 lbs coupled with the fact that the crew will not have to move to the CSM every time a corrective thruster firing is conducted makes passive or near passive precession control an attractive alternative.

Acknowledgement

Discussions concerning several aspects of this work with B. D. Elrod and W. W. Hough were especially helpful.

The able programming assistance of Mrs. P. R. Dowling is appreciated.

R J Ravera

1022-RJR-cf R. J. Ravera

Attachments
Figure 1-9

REFERENCES

1. Voelker, L. E., "Influence of Vehicle Dynamics on the Artificial Gravity Experiment on the Second Saturn Workshop," Bellcomm TM 70-1022-5, April 17, 1970.

2. Ravera, R. J., Hough, W. W., "Estimate of Daily Fuel Required to Control Gravity Gradient and Solar Precession during an Artificial G Experiment," Bellcomm Memorandum for File, June 15, 1970.

3. Elrod, B. D., "Solar Pointing Variations in Earth Orbit and the Impact on Mission Design," Bellcomm TR-70-1022-1, February 11, 1970.